INTERMEDIATE MAGNETOACOUSTIC WAVES

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Abstract. To understand the problem of solar coronal heating, role of fast-mode and slow-mode magnetoacoustic waves has been discussed from time to time by using the dispersion relation ω(*k*) = 0 (Porter et al. 1994; Kumar et al. 2006; Pandey and Dwivedi 2007; Chandra et al. 2010) derived a dispersion relation as a polynomial in ω and discussed about the fast-mode and slow-mode magnetoacoustic waves. Pekünlü et al. (2002) derived a dispersion relation as a polynomial in *k* and discussed about the fast-mode and slow-mode magnetoacoustic waves. When we expressed the said dispersion relation in ω as a polynomial in *k*, we obtained intermediate waves, besides the fast-mode and slow-mode waves. In the present investigation, we discuss the effects of density, magnetic field, temperature and the angle of propagation on the intermediate waves. However, we could not assign a physical significance of the intermediate waves yet.

Key words: solar coronal heating – magnetoacoustic waves.

1. INTRODUCTION

The temperature of solar corona (within $1-2\,R_{\text{Sun}}$) is maintained at the order of 10^6 K and the problem of solar coronal heating is yet unsettled. Here R_{Sun} denotes the solar radius. Porter et al. (1994), Pekünlü et al. (2002), Kumar et al. (2006), Pandey and Dwivedi (2007), and others investigated the problem of solar coronal heating through MHD waves. A common approach in all these investigations has been to derive a dispersion relation $\omega(k)=0$, from the basic MHD equations. After getting the dispersion relation, we can approach in two ways: (i) k is real and the roots are of the form $k_{\text{r}}+ik_{\text{i}}$, and (ii) ω is real and the roots are of the form $k_{\text{r}}+ik_{\text{i}}$.

Kumar et al. (2006) and Chandra et al. (2010) derived a fifth degree polynomial in ω and, for the real values of k, they discussed about the fast-mode and slow-mode magnetoacoustic waves. We mention the fact that Pandey and Dwivedi (2007), for the same set of MHD equations, derived a six degree polynomial in ω, but Chandra et al.

(2010) showed categorically that Pandey and Dwivedi (2007) unnecessarily introduced one additional root in the dispersion relation. Moreover, the five roots of both polynomials in ω are the same.

In the present investigation, we expressed the said dispersion relation as a polynomial in k. For the real values of ω from this dispersion relation, we got three types of waves: fast-mode, slow-mode and intermediate magnetoacoustic waves. The details of these intermediate waves are discussed here. In the next section we shall first derive the dispersion relation as a polynomial in ω and then rearrange the same as a polynomial in k.

2. DISPERSION RELATION

The basic equations for the present investigation are the following (Porter et al. 1994; Kumar et al. 2006; Chandra et al. 2010):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0\,, \tag{1}$$

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho(\vec{v}.\nabla)\vec{v} = -\nabla p + \frac{1}{4\pi}(\nabla \times \vec{B}) \times \vec{B} - \nabla \cdot \Pi\,, \tag{2}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B})\,, \tag{3}$$

$$\frac{Dp}{Dt} + \gamma p(\nabla \cdot \vec{v}) = (\gamma - 1)\left[Q_{\rm th} + Q_{\rm vis} - Q_{\rm rad}\right], \tag{4}$$

$$p = \frac{2\rho k_B T}{m_p}\,. \tag{5}$$

The symbols have their usual meanings, whereas the quantities $Q_{\rm th}$, $Q_{\rm vis}$ and $Q_{\rm rad}$ respectively are (e.g., Kumar et al. 2006):

$$Q_{\rm th} = k_{\|}\left(\frac{\partial T}{\partial z}\right)^2 T^{-1}, \qquad Q_{\rm vis} = \frac{\eta_0}{3}(\nabla \cdot \vec{v})^2, \qquad Q_{\rm rad} = n_e n_H Q(T)\,,$$

where $k_{\|}$ represents the conductivity along the magnetic field and is expressed by $k_{\|} \approx 10^{-6} T^{5/2}$. For this set of equations (1)–(5), the dispersion relation is (Kumar et al. 2006; Chandra et al. 2010):

$$\omega^5 + iA\omega^4 - B\omega^3 - iC\omega^2 + D\omega + iE = 0\,, \tag{6}$$

in which

$$A = c_0 + \frac{\eta_0}{3\rho_0}(k_x^2 + 4k_z^2)\,,$$

$$B = \frac{c_0\eta_0}{3\rho_0}(k_x^2 + 4k_z^2) + (c_s^2 + v_A^2)k^2\,,$$

$$C = \frac{3\rho_0}{\rho_0}c_s^2 k_x^2 k_z^2 + \frac{c_0 p_0}{\rho_0}k^2 + v_A^2 c_0 k^2 + \frac{4\eta_0 v_A^2 k_z^2 k^2}{3\rho_0}\,,$$

$$D = \frac{3c_0 p_0 \eta_0 k_x^2 k_z^2}{\rho_0^2} + \frac{4\eta_0 c_0 v_A^2 k_z^2 k^2}{3\rho_0} + v_A^2 c_s^2 k_z^2 k^2\,,$$

$$E = \frac{v_A^2 c_0 p_0 k_z^2 k^2}{\rho_0}\,,$$

and

$$c_0 = (\gamma - 1)k_\parallel k_z^2 \frac{T_0}{p_0}, \qquad v_A = \frac{B_0}{\sqrt{4\pi\rho_0}}, \qquad \eta_0 = 10^{-16}T_0^{5/2}, \qquad c_s^2 = \gamma p_0 / \rho_0\,.$$

Though Pandey and Dwivedi (2007) obtained a sixth degree polynomial for the dispersion relation for the same set of basic equations, Chandra et al. (2010) have found that the relation (6) is common for both cases. This dispersion relation is expressed as a polynomial in ω and, for the real values of k, five roots of the form $\omega_\mathrm{r} + i\omega_\mathrm{i}$ can be obtained.

The obtained five roots are of the form: $-i\omega_{1\mathrm{i}}; \pm\omega_{2\mathrm{r}}; \pm i\omega_{2\mathrm{i}}; \pm\omega_{3\mathrm{r}}; \pm i\omega_{3\mathrm{i}}$. The pure imaginary root corresponds to the thermal wave, whereas the other two complex roots correspond to the fast-mode and slow-mode waves. The real part of the root for the fast-mode wave is larger than that for the slow-mode wave. The real and imaginary parts of ω respectively give the frequency of the wave and its damping rate.

In Fig. 1 we have plotted the damping rate versus wavenumber k : panels (1a) and (1b) respectively correspond to slow-mode wave and fast-mode wave for $n_0 = 1010\,\mathrm{cm}^{-3}$, $B_0 = 100\,\mathrm{G}$, and $T_0 = 2\times 106\,\mathrm{K}$. In these two panels, we have considered the values $\theta \in \{15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ\}$. For the slow-mode waves, we found that as the value of k increases, after a certain value, called k_c, the slow mode wave vanishes. This feature is a bit different from that shown by Kumar et al. (2006). For the fast mode waves the graph is similar, as shown by Kumar et al. (2006) in their Fig. 1b.

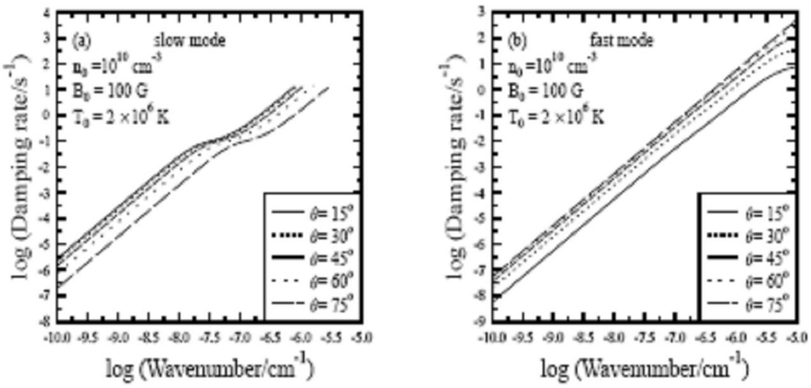

Fig. 1 – Variation of damping rate as a function of wavenumber for: (a) slow-mode wave and (b) fast-mode wave. Values of parameters are written there.

3. TRANSFORMATION OF DISPERSION RELATION AS A POLYNOMIAL IN *k*

Equation (6) can also be expressed as a polynomial in k as the following. By

$$\mu = \cos\theta, \quad c_0' = (\gamma - 1) k_{\parallel} T, \quad k_x^2 = (1-\mu^2)k^2, \quad k_z^2 = \mu^2 k^2 ,$$

the dispersion relation (6) can be rearranged as

$$A'k^6 + B'k^4 + C'k^2 + D' = 0 \tag{7}$$

where

$$A' = \frac{3c_0'\eta_0(1-\mu^2)\mu^4\omega}{\rho_0^2} + \frac{4c_0'\eta_0 v_A^2\mu^4\omega}{3\rho_0 p_0} + \frac{ic_0' v_A^2\mu^4}{\rho_0},$$

$$B' = v_A^2 c_s^2 \mu^2\omega - \frac{c_0'\eta_0(1+3\mu^2)\mu^2\omega^3}{3\rho_0 p_0} +$$
$$+ i\left[-\frac{3\eta_0 c_s^2(1-\mu^2)\mu^2\omega^2}{\rho_0} - \frac{4\eta_0 v_A^2\mu^2\omega^2}{3\rho_0} - \frac{c_0'\mu^2\omega^2}{\rho_0} - \frac{c_0' v_A^2\mu^2\omega^2}{p_0} \right],$$

$$C' = -(v_A^2 + c_s^2)\omega^3 + i[\frac{\eta_0(1+3\mu^2)\omega^4}{3\rho_0} + \frac{c_0'\mu^2\omega^4}{p_0}],$$

$$D' = \omega^5 .$$

For equation (7), ω is taken as a real number, and the roots obtained are of the form $\pm(k_{1r} + ik_{1i}) \pm (k_{2r} + ik_{2i})$ and $\pm(k_{3r} + ik_{3i})$. The results show that there are three magnetoacoustic waves and no thermal wave. The real and imaginary parts of the wavenumber are related to wavelength λ and damping length D as

$$\lambda = 2\pi / k_r, \qquad D = 2\pi / k_i .$$

For $n_0 = 10^{10}\,\text{cm}^{-3}$, $B_0 = 100$ G, $T_0 = 2\times10^6$ K, $\theta = 45^\circ$, and various values of ω, ranging from $10^{-0.4}\,\text{s}^{-1}$ to $10^{4.6}\,\text{s}^{-1}$, the roots of equation (7) are obtained. The real values k_r and imaginary values k_i are shown in Fig. 2 (a) and (b), respectively.

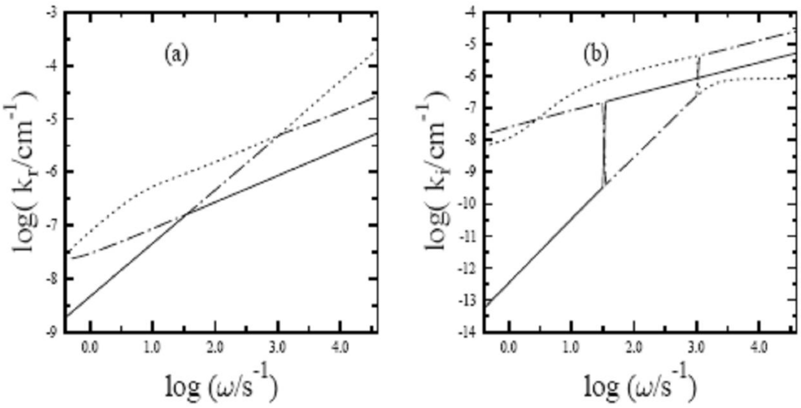

Fig. 2 – Real (a) and imaginary (b) values of the roots of equation (7) as functions of ω. Solid line: fast-mode wave; dashed line: slow-mode wave; dot-dashed line: intermediate wave.

Fig. 2 (a) shows that the wavelength of the intermediate wave is oscillating between the wavelengths of fast-mode and slow-mode waves. The wavelength of intermediate wave is equal to that of the fast-mode wave when $\omega \approx 10^{1.5}\,\text{s}^{-1}$ and to that of the slow-mode wave when $\omega \approx 10^3\,\text{s}^{-1}$.

Fig. 2 (b) shows that the damping length for slow-mode wave is smaller than that of fast-mode wave for $\omega < 10^3\,\text{s}^{-1}$. At $\omega \approx 10^3\,\text{s}^{-1}$, there is a fast increase in the damping length for slow-mode wave and then the damping length for slow-mode wave becomes larger than that for the fast-mode wave. Around $\omega \approx 10^{1.5}\,\text{s}^{-1}$, the damping length for the fast-mode wave decreases rapidly and the damping length for the intermediate wave increases rapidly. For $\omega > 10^{1.5}\,\text{s}^{-1}$, the damping length for intermediate wave becomes larger than that of fast-mode, as well as slow-mode waves up to $\omega \approx 10^3\,\text{s}^{-1}$. At this value, the damping length for the intermediate wave decreases rapidly, and becomes smaller than that of the fast-mode, as well as slow-mode waves. This shows that $\omega \approx 10^{1.5}\,\text{s}^{-1}$ and $\omega \approx 10^3\,\text{s}^{-1}$ are specific values.

3.1. VARIATION WITH DENSITY

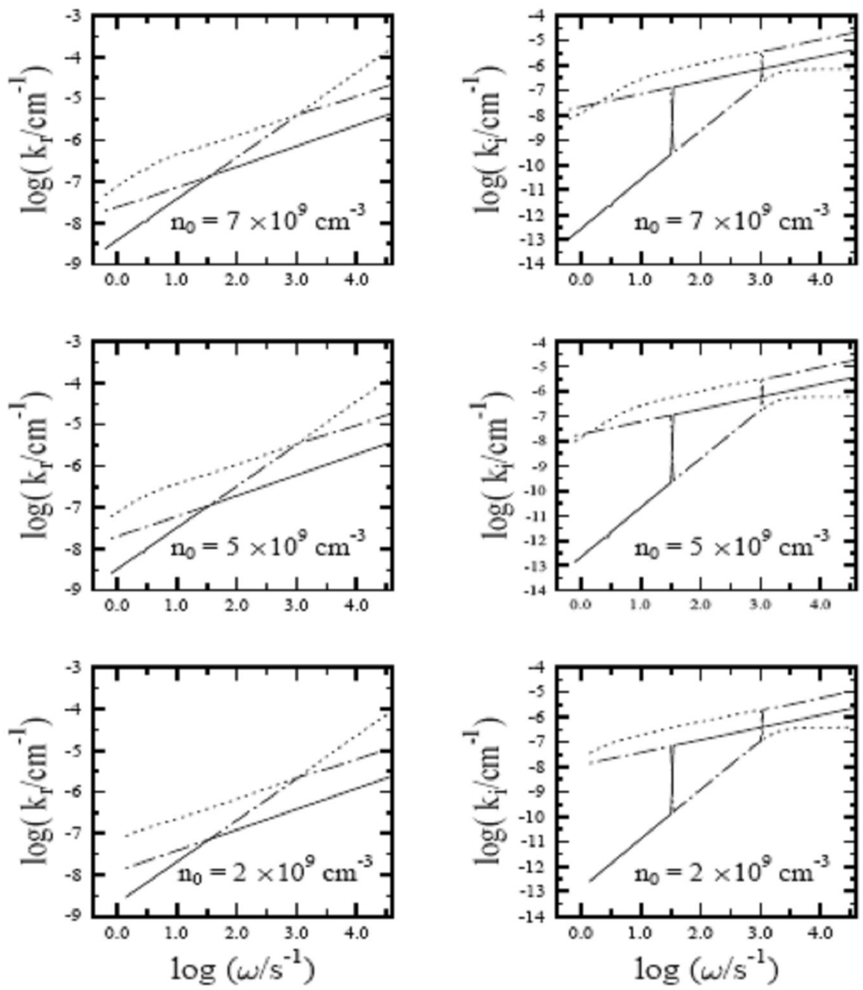

Fig. 3 – Same as Fig. 2, but for $n_0 = 2\times10^9$, 5×10^9 and 7×10^9 cm^{-3}.

In Fig. 3, $B_0 = 100$ G, $T_0 = 2\times10^6$ K, $\theta = 45^\circ$, but $n_0 = 2, 5, 7(\times10^9)\ \mathrm{cm}^{-3}$. The figure shows that it is not possible to make calculations for lower values of ω. The minimum value of ω up to which the calculations can be made increases with the decrease of density. Though there is change in the quantitative behavior with density, the qualitative behavior of results remains unaffected.

3.2. VARIATION WITH MAGNETIC FIELD

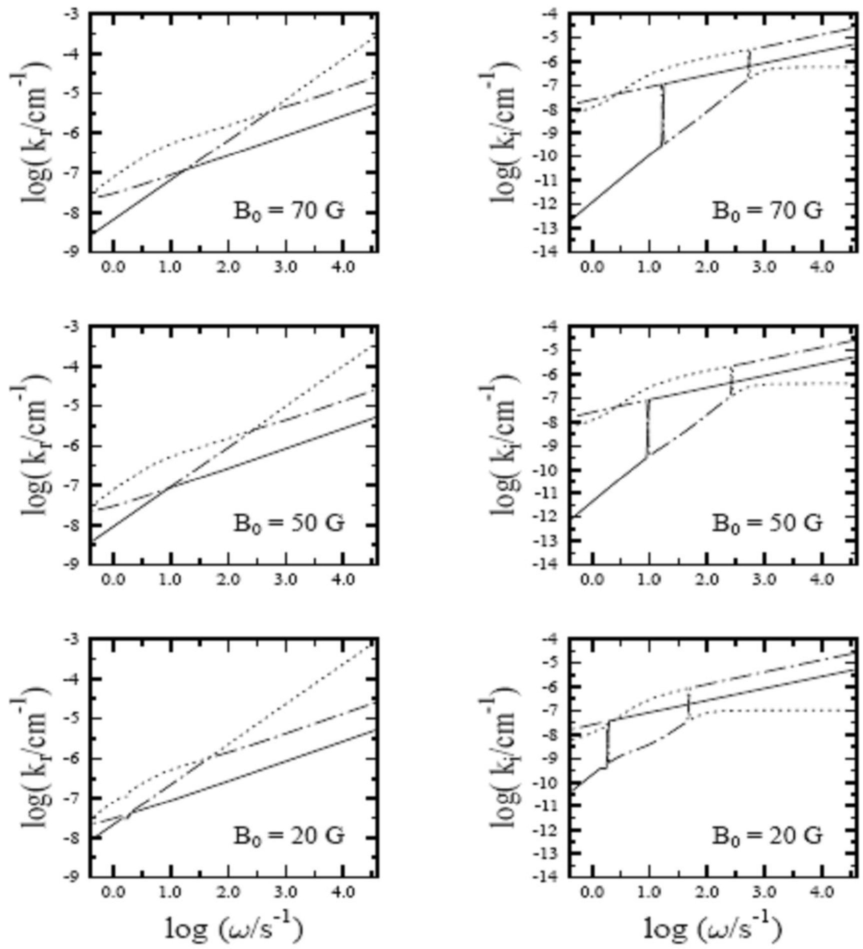

Fig. 4 – Same as Fig. 2, but for B_0 = 20, 50 and 70 G.

In Fig. 4, $n_0 = 2\times10^{10}\ \mathrm{cm}^{-3}$, $T_0 = 2\times10^6$ K, $\theta = 45^\circ$, but B_0 = 20, 50, 70 G. Here also the calculations are possible for the considered range of ω. The qualitative behavior of results remains unaffected, although the quantitative behavior changes with the magnetic field.

3.3. VARIATION WITH TEMPERATURE

In Fig. 5, $n_0 = 2\times10^{10}\ \text{cm}^{-3}$, $B_0 = 100$ G, $\theta = 45^\circ$, but $T_0 = 3, 4, 5(\times10^6)$ K. The figure shows that here also it is not possible to make calculations for lower values of ω. The minimum value of ω up to which the calculations can be made increases with the increase of temperature. Though there is change in the quantitative behavior, the qualitative behavior of results remains unaffected.

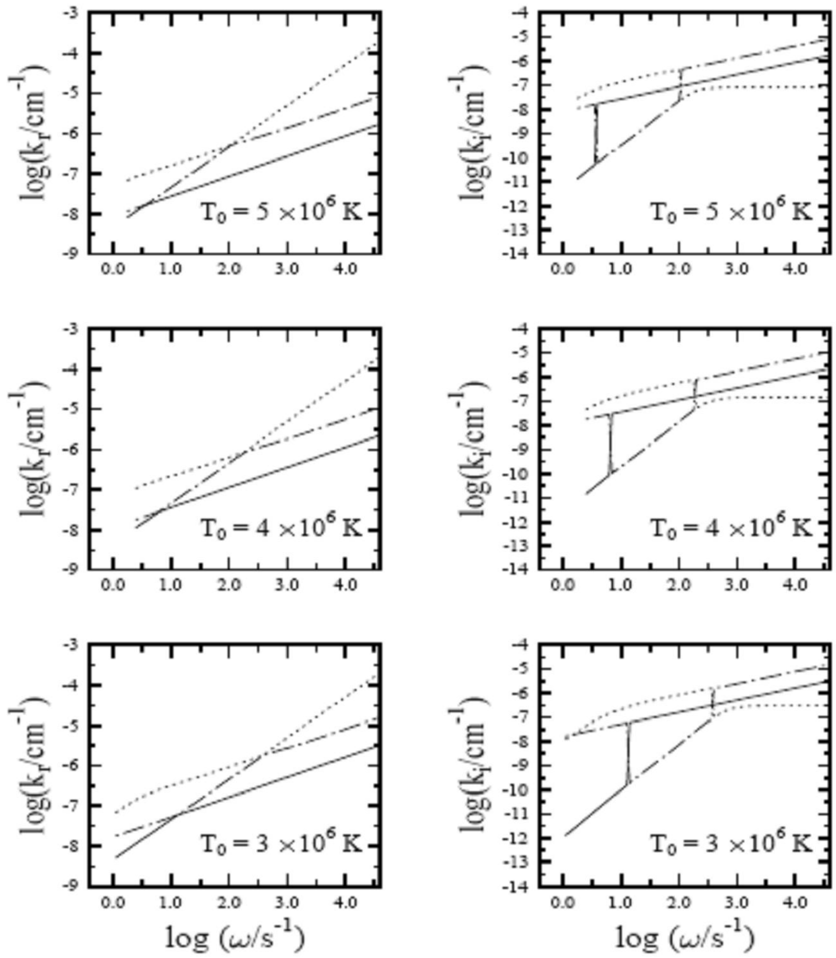

Fig. 5 – Same as Fig. 2, but for $T_0 = 3\times10^6$, 4×10^6 and 5×10^6 K.

3.4. VARIATION WITH ANGLE

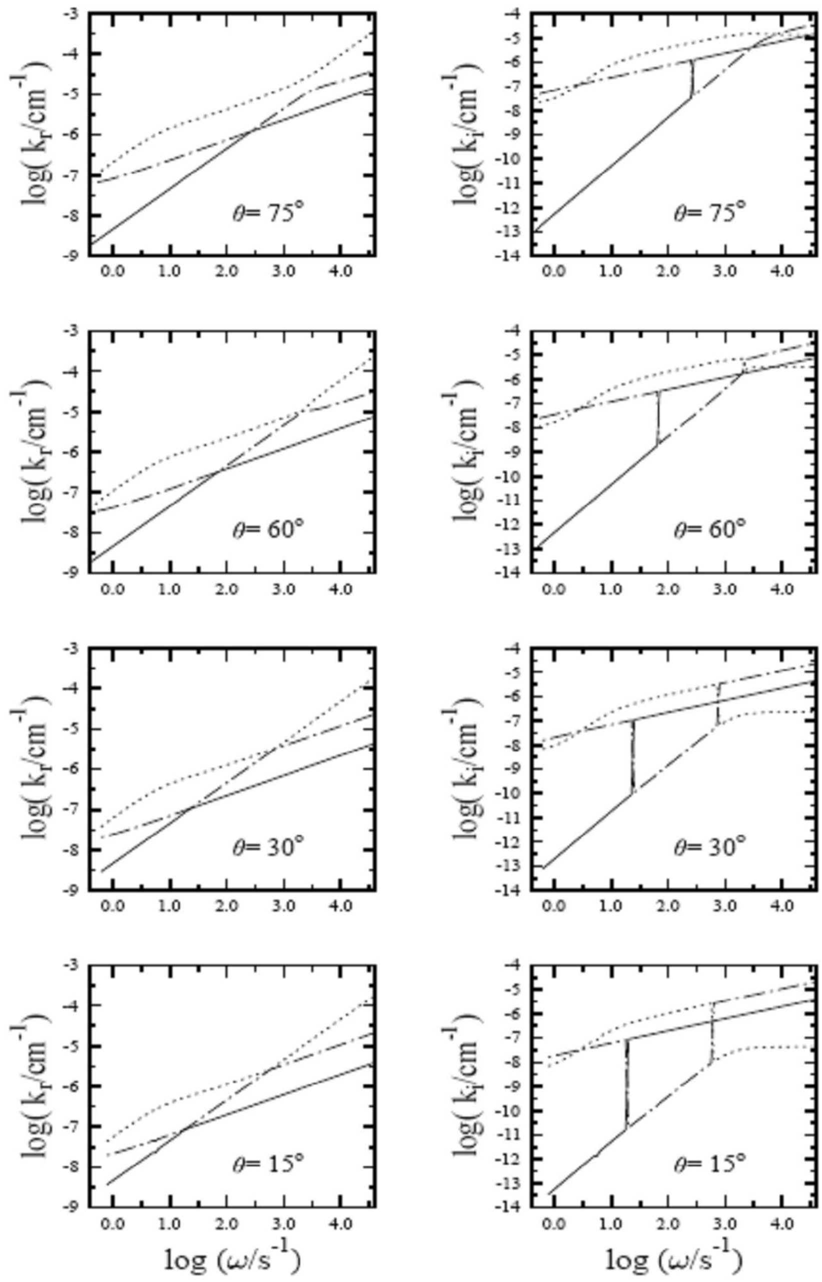

Fig. 6 – Same as Fig. 2, but for $\theta = 15^{o}$, 30^{o}, 60^{o} and 75^{o}.

In Fig. 6, $n_0 = 2\times10^{10}\ \mathrm{cm}^{-3}$, $B_0 = 100$ G, $T_0 = 3\times10^6$ K, $\theta = 15^\circ, 30^\circ, 60^\circ, 75^\circ$. For large values of ω, the wavelength of intermediate wave never becomes equal to that the slow-mode wave. The dissipation length is found to be very sensitive for the angle θ between the directions of propagation of the wave and the magnetic field.

3.5. VARIATION OF ω_1 AND ω_2 WITH MAGNETIC FIELD AND TEMPERATURE

Figs. 3–5 show that for one value (ω_1) of ω the intermediate wave coincides with the fast-mode wave. Further, at another value (ω_2) of ω, the intermediate wave coincides with the slow-mode wave. Fig. 3 shows that ω_1 and ω_2 do not depend on the density number density, and they are approximately 1.5 and 3.0, respectively. By virtue of Figs. 4–5, ω_1 and ω_2 depend on the magnetic field and temperature.

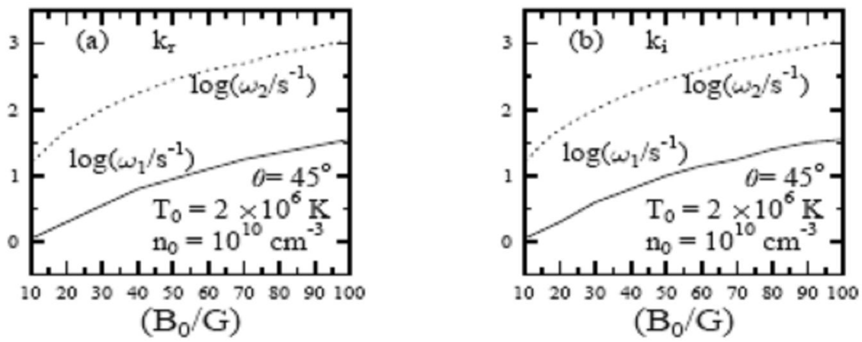

Fig. 7 – ω_1 and ω_2 as a function of B_0. Solid line: ω_1; dotted line: ω_2.

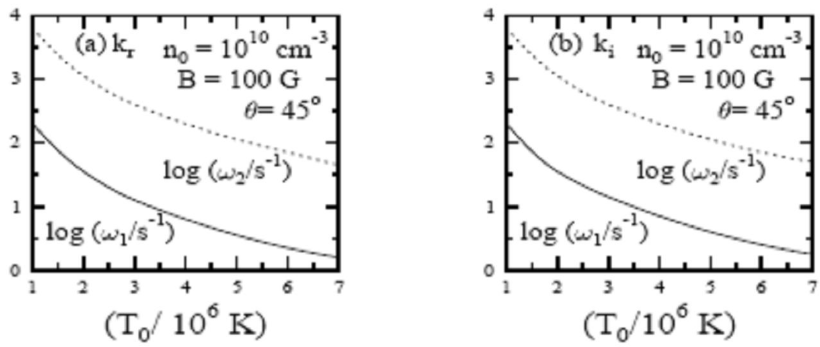

Fig. 8 – ω_1 and ω_2 as a function of T_0. Solid line: ω_1; dotted line: ω_2.

Fig. 7 shows the variation of ω_1 and ω_2 with the magnetic field B_0, for given values of n_0, T_0 and θ (see figure). Panel (a) is derived corresponding to the values of $k_{\rm r}$; panel (b) is derived corresponding to the values of $k_{\rm i}$.

Fig. 8 shows the variation of ω_1 and ω_2 with the temperature T_0, for given values of n_0, B_0 and θ (see figure). Panel (a) is derived corresponding to the values of $k_{\rm r}$; panel (b) is derived corresponding to the values of $k_{\rm i}$.

4. CONCLUSIONS

It is interesting to note that, when the dispersion relation was expressed as a polynomial in k, besides the fast-mode and slow-mode waves, we got additional wave, called the intermediate wave, whose phase velocity has always been in between those of the fast-mode and slow-mode waves. From Fig. 2 we found that, for $\omega > 10^3\,{\rm s}^{-1}$, the dissipation length for slow-mode wave becomes smaller than that of the fast-mode wave. If the value of the magnetic field increases, the values of ω_1 and ω_2 also increase. If the value of the temperature increases, the values of ω_1 and ω_2 decrease.

Here we have discussed the effects of density, magnetic field, temperature and angle of propagation. The most significant change has been found for the dependence of the angle between the direction of propagation of wave and the magnetic field.

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